ZACHARY

Galvanizing Growth Solution

$$a_{n1} = 125e\hat{}(-c_n/100)$$
$$a_{n2} = 5e\hat{}(-c/5)$$
$$g_{m} = \left\lfloor a_{n1} S_n \left(\frac{3-2\sqrt{2}\,\mathrm{erfc}^{-1} r}{200}\right)\left(1-\frac{S_n}{P_n}\right)\right\rfloor$$
$$g_{ab} = \left\lfloor a_{n2} S_b \left(\frac{3-2\sqrt{2}\,\mathrm{erfc}^{-1} r}{200}\right)\left(1-\frac{S_a}{P_a}\right)\right\rfloor$$
$$x = 5 - 3\sqrt{2}\,\mathrm{erfc}^{-1} r$$
$$y = 11 - 5\sqrt{2}\,\mathrm{erfc}^{-1} r$$
$$i_n = S_n + 10\left(\left|\frac{P_n}{S} - S_n\right| + \frac{P_n}{S} - S_n\right)$$

Definitons
c = expenditures
g = growth
s = currently sick
i = cost (direct / impact)
x = inc. period
y = recc period

steps:
1. deal with an outbreak
2. cost per case
3. imposing a quarantine
4. setting statistics

Part 1.
$\boxed{\text{Dealing with an outbreak}}$

1. each S gives $x a_{n1}$ new S's.

$$\log_a b = \frac{\ln b}{\ln a}$$

$$g\text{-rate} = (x a_{n1} - 1)^{\frac{1}{x}}$$

so that a_{n1} need only be less than $\frac{1}{x}$.

for this,

$$125e\hat{}(-c_n/100) < \frac{1}{x}$$
$$e\hat{}(-c_n/100) < \frac{1}{125x}$$
$$-c_n/100 < \ln\left(\frac{1}{125x}\right)$$

$\ln(\frac{1}{2}) \div \ln(a)$

$$c_n > 100\ln\left(\frac{1}{125x}\right)$$

consider that the outbreak lasts

$\log(a+b)$

<half life> $\times(\log_2 n + 1)$ n is # cases at start

and <half life> $= \ln(\frac{1}{2}) \div \ln(g\text{-rate})$

so that the outbreack lasts <duration>

$\cancel{\ln(\tfrac{1}{2}) \div \ln[125 e\hat{}(-c_n/100) - 1] \div x \times \ln(n) \div \ln(2)}$,

costing c × <duration> + sum(i)

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1<cont.>: cost of an outbreak is given by

$$c \times \ln(\tfrac{1}{2}) \times \ln(n) \div \ln\left(\left(125\,x e^{\wedge}(-c_n \div 100) - 1\right)^{\wedge}(1 \div x)\right) \div \ln(2)$$

let $a = \ln(125\,x e^{\wedge}(-c_n \div 100) - 1)^{\wedge}(-1 \div x)$

add to the cost $\quad \operatorname{ReLU} x = \frac{|x| + x}{2}$

$$(+)\ \underbrace{y\,a}_{\text{part 2}} + \underbrace{\operatorname{ReLU}\left[\left(y - \tfrac{p}{5}\right) y(a-1)^{-1} - \ln\left(\tfrac{5y}{p}\right) \div \ln(a) \times \tfrac{p}{5}\right]}_{\text{part 3}} \times 20$$

Now, consider that

$$\frac{\partial\langle cost\rangle}{\partial c} = 0$$ the derivative of that function is

$$\left(\ln(\tfrac{1}{2}) \times \ln(n) \div \ln(2) + y\right) \times \frac{\partial}{\partial x}(\ln a)$$

$$+ \frac{\partial}{\partial x} \operatorname{ReLU}\left(\left(y - \tfrac{p}{5}\right) y(a-1)^{-1} - \ln\left(\tfrac{5y}{p}\right) \div \ln(a) \times \tfrac{p}{5}\right)$$

let $f(x) = a$ when $c = x$

$$\frac{\partial}{\partial x} \ln f(x) = \ln' f x \cdot f' x = \frac{f' x}{f x}^{*} = -\frac{1\,e^{-c}}{100\,e^{\wedge}(\frac{-c}{100})}$$

$f' x = -1.25 x e^{-c}$

$^{*} \ln' x = x^{-1}$

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The derivative of $ReLU(g(x))$ is $ReLU'(g(x))g'(x)$

$ReLU'$ is 0 for negative and 1 for positive, undefined at 0.
In other words (maths): $\frac{|x| \div 2x + 1}{2}$

$$g(x) = \left(y - \frac{p}{5}\right) y \left(a^{-1} - 1\right)^{-1} - \ln\left(\frac{5y}{p}\right) \div \ln(a) \times \frac{p}{5}$$

let $h(x) = (x^{-1} - 1)^{-1}$
let $i(x) = x^{-1}$

$$g'(x) = y\left(y - \frac{p}{5}\right) \cdot \frac{\partial}{\partial x}\left(h(f(x))\right) + \ln\left(\frac{5y}{p}\right) \times \frac{\partial}{\partial x}\left(i(f(x))\right) \times \frac{p}{5}$$

now, $f'(x) = -1.25xe^{-c}$,

$h'(x) = (x-1)^{-2}$, and $i'(x) = -x^{-2}$

So that then, the full derivative is

$$g'(x) = y\left(y - \frac{p}{5}\right) \times \left(\left(125xe\text{^}(-c \div 100) - 1\right)\text{^}(1 \div x) - 1\right)\text{^}{-2} \times 1.25xe^{-c}$$

$$+ \ln\left(\frac{5y}{p}\right) \times \frac{p}{5} \div 1.25x \times e^{c} = 0$$

$$e^{c} \times \ln\frac{5y}{p} \times \frac{p}{5} \div 1.25x = y\left(y - \frac{p}{5}\right) \times 1.25xe^{-c} \times (f(x) - 1)^{-2}$$

$$e^{2c} \times (f(x) - 1)^{2} = 1.25xy\left(y - \frac{p}{5}\right)\frac{5}{p} \div \ln\left(\frac{5y}{p}\right)$$

let $j(x) = e^{2x}$, $k(x) = (f(x) - 1)^{2}$

let $z = 1.25xy\left(y - \frac{p}{5}\right)\frac{5}{p} \div \ln\left(\frac{5y}{p}\right)$

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$z = j(c) \cdot k(c)$ $\qquad j^{-1}(x) = -100\ln(x)$

$\frac{z}{j(c)} = k(c)$

$c = j^{-1}\left(\frac{z}{k(c)}\right)$

let $l(x) = zx^{-1}$

$c = k^{-1} \circ l \circ j(c)$

$c = -100\left(\ln(2) + 2\ln(\$(x)+1)\right)$

~~$e^c = -100z \cdot \left((125xe^\wedge(-c \div 100) - 1)^\wedge(1-x) + 1\right)$~~

$e^c + 100z = -100z((125xe^\wedge(-c \div 100$

~~what if we did $c = k^{-1} \circ j(c)$ instead?~~

$c = 1.25xe^\wedge(z \div -100\ln c)$

$\ln(c) = \ln(1.25x) + z \div 100\ln c$

$\ln c \ln\left(\frac{c}{1.25x}\right) = \frac{z}{100}$

let $p = \ln c$, $\ln 1.25x = q$, $r = \frac{-z}{100}$

Cont. from bottom

$p^2 + pq + r = 0$

$p = \frac{-q \pm \sqrt{q^2 - 4r}}{2}$

$\boxed{\text{we want the high c}}$

$$\boxed{c = e^\wedge\left(\frac{-q + \sqrt{q^2-4r}}{2}\right)}$$

Part 2.

G. G. Sol. pg. V Cost Per Case

So far, we have c (expenditures daily) as

$$e^\wedge\left(-\frac{5x}{8}-\sqrt{\frac{25x}{4}+\frac{z}{100}}\right) \text{ where}$$

$$z=\left(\frac{1.25xy^2}{p}-1.25xy\right)\div\ln\left(\frac{5y}{p}\right)$$

Remember, net cost of an outbreak is

$$c\cdot\ln\tfrac{1}{2}\cdot\ln n\div\ln fc\div\ln 2+yf(c)$$
$$+\mathrm{ReLU}\left(\frac{y(5y+p)}{5(fc-1)}-\ln\frac{5y}{p}\cdot\frac{p}{5}\div\ln fc\right)$$

n (num cases) is found in $\ln n$, and all four instances of fc.

Now, to get the cost per additional case, required to go to part 3 (travel), we take the derivative again, this time in terms of n and not c. We begin with

$\boxed{\text{let } f^*(x)=f(x)\div n,\ (125xe^\wedge(-c\div 100)-1)^\wedge(-1\div x)}$

derivative: $c\ln\frac{1}{2}\div\ln 2\,\frac{3}{8x}\ln fc+yf^*c+\frac{3}{8x}\mathrm{ReLU}$ ~~~~

Integrate

Integrate by parts: $\frac{3}{8x}\ln fc=\ln' fc\cdot f'c$.

f' in terms of n is simply f^*. $\ln'$ is x^{-1}.

We set: $\dfrac{f^*c}{fc}$ remember that f^* is $\dfrac{fc}{n}$.

Simplifying, we set n^{-1}.

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We now have $\frac{\partial}{\partial x} \ln fc$ is n^{-1}.

Consider the ReLU component.

Using u-substitution, it's derivative is

$\text{ReLU}'(u) \frac{\partial}{\partial n} u$. ReLU' is 0 if negative, else 1.

the statement is either $\emptyset$ or $\frac{\partial}{\partial n}\left(\frac{y(5y+p)}{5(fc-1)} - \frac{p}{5} \ln \frac{5y}{p} \div \ln fc\right)$

...equivalent to $\text{ReLU}\left(\frac{\partial}{\partial x} m\right)$! Integrating ReLU is easy!

Integrate by parts: $\frac{\partial}{\partial n} \frac{y(5y+p)}{5(fc-1)} - \frac{p}{5} \ln \frac{5y}{p} \frac{\partial}{\partial n} (\ln fc)^{-1}$

First, we have $\frac{y(5y+p)}{5} \frac{\partial}{\partial n} (nf^{*}c-1)^{-1}$

(remember, $fc = nf^{*}c$.)

Using u-substitution, we have $\frac{\partial}{\partial n} u^{-1} = -u^{-2} \cdot \frac{\partial}{\partial n} u$
the derivative of u is clearly $f^{*}c$. the whole is now

$$\frac{y(5y+p)f^{*}c}{5(fc-1)^2} - \frac{p}{5} \ln \frac{5y}{p} \frac{\partial}{\partial x} (\ln fc)^{-1}.$$

Using u-substitution again, we get $-\ln fc^{-2} \cdot \frac{\partial}{\partial x} \ln \circ fc$.
Furthermore, $\frac{\partial}{\partial x} \ln \circ fc$ is $\ln' fc \cdot f'c$. f' is f^{*}, $\ln' x$ is x^{-1},
we get $\frac{-\ln fc^{-2}}{n}$, or $-1 \div ((\ln \circ fc)^2 n)$, or
$-(n \ln^2 nf^{*}c)^{-1}$

G-G. Sol- pg-VII - Part 3 Imposing Quarantine

We have the cost per case. It is

$$C \ln\tfrac{1}{2} \div \ln 2 \div n^2 + yf^*c + \mathrm{ReLU}\left(\frac{y(5y+p)f^*c}{5(nf^*c-1)} + \frac{1}{n\ln(2nf^*c)}\right)$$

note that n appears in several places, so that cost per case is dependent on the number of existing cases.

The effectiveness of stopping cases spreading is given by $5e\hat{}(-\varepsilon/5) = a_{n2}$. Plug into

$$g_{ab} = a_{a2} S_b \sim\!\sim\!\sim \left(1-\frac{S_a}{p_a}\right)$$

normal dist.
w. mean 0.015

The normal distribution can be replaced with it's mean, as we only care for the average. We then have:

$$g_{ab} = 5\cdot e\hat{}(-C_a/5)\cdot 0.015 \cdot S_b \cdot \left(1-\frac{S_a}{p_a}\right).$$ The cost to prevent a case is it's derivative,

$$\frac{3}{40}\left(1-\frac{S_a}{p_a}\right)\frac{\partial}{\partial C_a} e\hat{}(-C_a/5), \text{ or } -\frac{3}{200} e\hat{}(-C_a/5)\left(1-\frac{S_a}{p_a}\right)S_b$$

The degree of quarantine is thus when the cost to prevent a case equals that which the case would have, when

$$-\frac{3}{200} S_b\, e\hat{}(-C_a/5)\left(1-\frac{S_a}{p_a}\right) = C\ln\tfrac{1}{2} \div \ln 2 \div n^2 + yf^*c + \mathrm{ReLU}\sim.$$

$$\text{let } o = \left(C\ln\tfrac{1}{2} \div \ln 2 \div n^2 + yf^*c + \mathrm{ReLU}\sim\right) \div \left(-\frac{3}{200} S_b \left(1-\frac{S_a}{p_a}\right)\right)$$

6. 6. Sol. pg. VIII

We've just defined O as the stuff equivalent to e^(-Ca/5). We then have

$$e\hat{}(-Ca/5) = O,$$ and O is independent of Ca.

Solving, $-Ca/5 = \ln(O)$,

$$Ca = -5\ln O.$$ Expanding,

$$\boxed{Ca = \frac{-\frac{3}{200} S_b\left(1 - \frac{S_a}{pa}\right)}{4\ln\frac{2}{2} \div \ln 2 \div n^2 + y\int^x c + \mathrm{Re}|v\sim}}$$

Cont. from bottom

$\int x_0$ is made by a normal dist. w. $\mu = 1.5$, $\sigma = 1$.

Deviation is t^{-2} after t days. Multiply. Let m = current mean

PART FOUR

Getting Statistics.

y is easy. Recovery is a shifted case distribution, y is the number of days from patient 0 to recovery. But before recovery? This period is important.

The expected value of y is

$$\int_t^\infty t \cdot \text{normal distribution} = 1 - e^{-(t-\mu)^2/2\sigma^2} \cdot \frac{2t-\mu}{2\sqrt{2\pi}\,\sigma^3}$$

y has $\mu = 11$, $\sigma = 5$, and so this is $1 - \frac{2t-5}{e\hat{}((t-5)/(.189))\,54\sqrt{2\pi}}$

Continued on p IX.

Now, x. Consider the probability that a given growth rate came from a given x. If $\int^{-1}\alpha$ is not 5, it could be skewed. Estimate as

$$\frac{1}{2} = e^{-(x-m)^2 t/2 - (1.5-x)^2/2} \cdot \frac{\sqrt{t}}{2\pi}$$

6.6. Sol. p. IX

we have that $\frac{1}{2} = e^\wedge(-(x-m)^2 t/2 - (x-1.5)^2/2) \cdot \frac{\sqrt{t}}{2\pi}$
solving for x,

$$e^\wedge \sim\sim = \frac{\pi}{\sqrt{t}}$$

$$-(x-m)^2 t/2 - (x-1.5)^2/2 = \ln\frac{\pi}{t}$$

$$tx^2 + x^2 - 3x - 2mx + 2.25 + 3m + \ln\frac{\pi}{t} = 0$$

using quadratic formula, $a = t+1$, $b = -(3+2m)$, $c = \frac{9}{4} + 3m + \ln\frac{\pi}{t}$,

$$\boxed{x = \frac{3+2m \pm \sqrt{4m^2 - 3t(3+4m) + 4\ln\frac{t}{\pi}(t+1)}}{2(t+1)}}$$

We were also left with, at the end of p. VIII, the below equation for y. We get this because y is the mean, and so it's integral is half of its. Solving:

$$2\left(1 - \frac{2y-5}{e^\wedge \sim\sim}\right) = 1 - \frac{2t-5}{\sim\sim},$$

let the $1 - \sim$ stuff be $m(x)$.

$$2m(y) = m(t).$$

$$y = m^{-1}(2m(t)). \quad (e^\wedge - 972\sqrt{2\pi}(t-5)) \cdot (2t-5) = m(x)$$

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$$m^{-1}(x) = W_0^*(486\sqrt{2\pi}\, e^{2430\sqrt{2\pi}} x) \div 972\sqrt{2\pi}$$

$$\boxed{y = W_0\left(486\sqrt{2\pi}\, e^{486\sqrt{\pi}(15-2t)} \cdot (2t-5)\right) \div 972\sqrt{\pi}}$$

w is the Lambert W function